SOME RESULTS ON n-EDGE MAGIC LABELING –part 2

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Abstract: [2,3,4,5] defined and some results of 0-edge magic labeling and , [7] defined 1-edge magic labeling and [6] defined for n- edge magic labeling for path, cycle. In this paper extended n –edge magic to Ladder graph , Friendship graph, Armed Crown graph, G = $P_{t+1} \odot K_{1,t}$, Splitting graph, Prism Graph, Web graph. And find the order and size of n-edge magic labeling of graphs.

Keywords: n- Edge Magic Labeling, Ladder graph , Friendship graph, Armed Crown graph, G = $P_{t+1} \odot K_{1,t}$, Splitting graph, Prism Graph, Web graph

1. Introduction

Let G(V,E) be a simple , finite, undirected graph in this paper with order p and size q. Labeling is one-to-one mapping from the set of all vertices, or the set of all edges, or the set of all vertices and edges called vertex labeling, or an edge labeling, or a total labeling to integers. The origin of this labeling is introduced by Rosa in 1967. 0-Edge Magic Labeling is kind of Labeling for Some Class of graphs introduced by Jayapriya.J , Thirusangu, P Nedumaran [2,3,4,5] . Neelam Kumari, Seema Mehra introduced 1 edge magic labeling of path, cycle, double star in [7]. Later on same people introduced n edge magic labeling path, cycle, sun graph in 2013[6].

In this paper generalize the n- edge magic labeling results to Ladder graph , Friendship graph, Armed Crown graph, G = $P_{t+1} \odot K_{1,t}$, Splitting graph, Prism Graph, web graph and also find the order and size of n-edge magic labeling of graphs.

2. Preliminaries

An **edge-magic labeling** of a (p; q)-graph G is a bijective function f : V(G) ∪ E(G)→{1; 2; : : : ; p + q} such that f(u) + f(v) + f(uv)=k is a constant for any edge uv of G. In such a case, G is said to be edge-magic and k is called the valence of f.

0-Edge Magic Labeling: Let G = (V, E) be a graph where V = {v_i, 1 ≤ i ≤ n}, and E = {v_i v_{i+1},1 ≤ i ≤ n}. Let f : V→ {-1, 1}, and f * : E → {0}, such that all uv ϵ E, f *(uv) = f (u) + f (v) = 0 then the labeling is said to be 0- Edge Magic labeling.

A (p, q) graph G is said to be **(1,0) edge-magic** with the common edge count k if there exists a bijection f : V(G) → {1, …, p} such that for all e = uv ϵ E(G), f(u) + f(v) = k. It is said to be (1, 0) edge anti-magic if for all e = (u,v) ϵ E(G), f(u) + f(v) are distinct.

A (p,q) graph G is said to be **(0,1) vertex-magic** with the common vertex count k if there exists a bijection f: E(G) → {1, …, q} such that for each u ϵ V(G), e ϵ Σ f(e) =k for all e = uv ϵ E(G) with v ϵ V(G). It is said to be (0, 1) vertex-antimagic if for each u ϵ V (G), e ϵ Σ f(e) are distinct for all e = uv ϵ E(G) with v ϵ V(G).

Let G= (V, E) be a graph where V= {vi, 1 ≤ i ≤ t} and E = {v_i v_{i+1},1 ≤ i ≤ t-1}. Let f: V → {-1, 2} and f*: E → {1} such that for all uv Є E, f*(u v) = f (u) + f (v) =1 then the labelling is said to be **1-Edge Magic Labeling**.

A (p,q) graph G is said to be **(1,1) edge-magic** with the common edge count k if there exists a bijection f : V(G) ∪ E(G) → {1, …, p+q} such that f(u) + f(v) +f(e) = k for all e = uv ϵ E(G). It is said to be (1,1) edge-antimagic if f(u) + f(v) + f(e) are distinct for all e = uv ϵ E(G).

G_+ = G⊙K_1 is a graph obtained by joining exactly one pendant edge to every vertex of a graph G.

A **sun S_t** is a cycle on t vertices with an edge terminating in a vertex of degree 1 attached to each vertex on the cycle.

A **complete binary tree** T is a tree with a central vertex of degree 2 , all other vertices that are not leaves of degree 3, and all leaves at the same distance from the central vertex.

Let G=(V, E) be a graph where V= { vi, 1≤i≤ t } and E = {vi v i+1,1≤i≤t-1}.

Let f: V → {-1, n+1} and f*: E → {n} such that for all uv E, f*(u v) = f (u) + f (v) = n then the labeling is said to be **n-Edge Magic Labeling**.

The Ladder graph L_t is a planar undirected graph with 2n vertices and n+2(n-1) edges. The ladder graph can be obtained as the Cartesian product of two path graphs, one of which has only one edge.

The **Friendship graph** f_t is a collection of n triangles with a common vertex. It may be also pictured as a wheel with every alternate rim edge removed. The Generalised friendship graph $f_{m,t}$ is a collection of t cycle (all of order m), meeting at a common vertex.

The Generalised friendship graph is because of its shape, also referred to a Flower.

An **Armed Crown graph** $C_t \odot P_m$ is a connected graph in which path P_m is attached to every vertex of the cycle C_t.

Let G = (V, E) be a graph then **G = $P_{t+1} \odot K_{1,t}$** is a graph where u_i's and v_i's are vertices. v_i's are t pendent vertices to u_1.

Theorem1 Pt admits n-Edge Magic Labeling for all t.

Theorem2 Ct admits n-Edge Magic Labeling when t is even.

Theorem 3 A sun graph St is n-edge magic only when t is even.

Main Results

Theorem 4 If a Ladder graph L_t admits n-Edge Magic Labeling for all t

Proof

Let G = (V, E) be a graph where V = {v_i,1≤ i≤ t} and E = {v_i v $_{i+1}$, 1 ≤ i ≤ t-1}.

Let f: V → {-1, n+1}

Such that
$$f(v_i) = \begin{cases} -1\,,\ if\ i\ is\ odd, \\ n+1\,,\ if\ i\ is\ even. \end{cases} \qquad \text{for } 1 \le i \le t,$$
$$f(u_i) = \begin{cases} -1\,,\ if\ i\ is\ even, \\ n+1\,,\ if\ i\ is\ odd. \end{cases} \qquad \text{for } 1 \le i \le t,$$

we have
$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & if\ i\ is\ odd, \\ (n+1)+(-1) = n & if\ i\ is\ even. \end{cases}$$
$$f^*(u_i u_{i+1}) = \begin{cases} -1+(n+1) = n & if\ i\ is\ even, \\ (n+1)+(-1) = n & if\ i\ is\ odd. \end{cases}$$
$$f^*(v_i u_i) = -1 + (n+1) = n \qquad \text{for } 1 \le i \le t.$$

Hence L_t admits n-Edge Magic Labeling.

Next we define order and size of ladder n –edge magic graph.

Proof given by induction method.

Generalised form of ladder graph L_t is

Case 1:

If t = 1

Ladder Graph exist 2 vertices and 1 edge.

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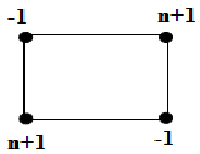
Let $v_1 = 1$ and $v_2 = n + 1$
Where v_1 and v_2 are adjacent to each other.
Therefore $f^*(v_1 v_2) = f(v_1) + f(v_2) = -1 + (n + 1) = n$.
Therefore L_t is n-Edge Magic Labeling Graph (Since t = 1).

Case 2:

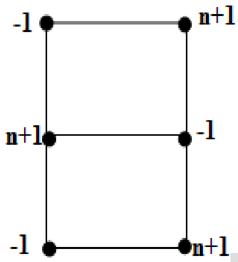
If t = 2



Ladder Graph exist 4 vertices and 4 edges.
Therefore L_t is n-Edge Magic Labeling Graph (Since t = 2).

Case 3:

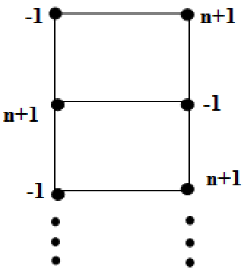
If t = 3



Ladder Graph exist 6 vertices and 7 edges.
Therefore L_t is n-Edge Magic Labeling Graph (Since t = 3).
Continuing this process, we get

Case n:

If t = n



Result: Therefore Ladder Graph is in the form of **2t** vertices with **3t-2** edges in n-Edge Magic Labeling for all t = 1, 2, 3,….., n.

Theorem 5 Let $G = f_{m,t}$ be a flower graph then G admits n-Edge Magic Labeling for all m > 3 and m is even then flower graph is in the form of **3n + 1 vertices** with **4n edges** for all n = 1,2,3,…n in n- edge magic labeling.

Proof

Let G = (V,E) be a flower graph denoted by $f_{m,t}$ and w_1 is a common vertex in $f_{m,t}$. Let v_i's and u_i's vertices in $f_{m,t}$.
Let $f: V \rightarrow \{-1, n + 1\}$
Such that

$f(w_1) = -1$
$f(v_i) = n+1$ for $1 \le i \le m$,
$f(u_i) = -1$ for $1 \le i \le t$,

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we have,

$$f^*(w_1 v_i) = -1 + (n+1) = n \quad \text{if } 1 \le i \le m,$$
$$f^*(v_i u_i) = (n+1) + (-1) = n \quad \text{if } 1 \le i \le m,\ 1 \le i \le t.$$

Hence the proof.

If $f_{m,t}$ be a n-Edge Magic Labeling then the order and size are discussed by induction Where t is collection of cycle meeting at a common vertex and all of order m.

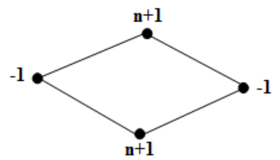
Flower Graph for n-Edge Magic Labeling exists only when m > 3 for all m is even.

ie).,
$$f_{m,t} = \begin{cases} m > 3, & \textit{for all m is even.} \\ t, & \textit{for all } t = 1,2,\ldots n. \end{cases}$$

Let fix m = 4

Case 1:

If m = 4 and t = 1



Flower Graph $f_{4,1}$ exist 4 vertices and 4 edges.

Let $w_1 = -1$, $v_1 = n + 1$, $v_2 = n + 1$, $u_1 = -1$

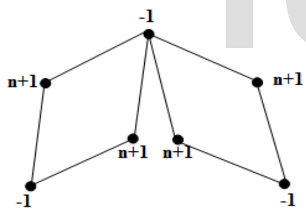
w_1 and v_1, w_1 and v_2 , v_1 and u_1, v_2 and u_1 are adjacent to each other.

Therefore
$$f^*(w_1 v_1) = -1 + (n+1) = n,$$
$$f^*(v_1 u_1) = (n+1) + (-1) = n,$$
$$f^*(u_1 v_2) = -1 + (n+1) = n,$$
$$f^*(v_2 w_1) = (n+1) + (-1) = n.$$

Therefore $f_{m,t}$ exists n-Edge Magic Labeling (Since m = 4 and t = 1).

Case 2:

If m = 4 and t = 2

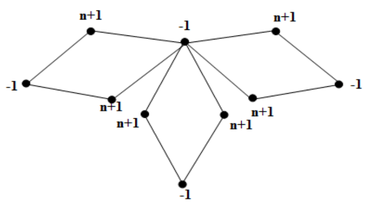


Flower Graph $f_{4,2}$ exists 7 vertices and 8 edges.

Therefore $f_{m,t}$ exists n-Edge Magic Labeling (Since m = 4 and t = 2).

Case 3:

If m = 4 and t = 3



Flower Graph $f_{4,3}$ exists 10 vertices and 12 edges.

Therefore $f_{m,t}$ exists n-Edge Magic Labeling (Since m = 4 and t = 3).

Continuing this process, we get

Case n:

If m = 4 and t = n

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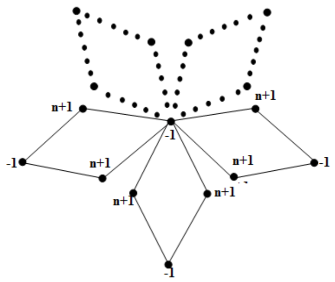


Therefore Flower Graph $f_{m,t}$ is in the form of m = 3n + 1 (where n = 1) and t = n (for all n = 1,2,…n).

Therefore Flower Graph for n-Edge Magic Labeling is in the form of **3n + 1 vertices** with **4n edges** for all n = 1,2,3,…n.

Theorem 6 An Armed Crown graph $C_t \odot P_m$ admits n-Edge Magic Labeling when t is even then in the form of 6n vertices and 6n edges for all n = 1,2,3,……n in n-edge magic labeling.

Proof

Let $v_1, v_2, \ldots\ldots\ldots v_t$ be the vertices of cycle C_t and $u_1, u_2, \ldots\ldots\ldots u_m$ be the vertices of each edge attached to $v_1, v_2, \ldots\ldots\ldots v_t$ and $w_1, w_2 \ldots\ldots\ldots w_m$ be the end vertices of each edge attached to $u_1, u_2 \ldots\ldots\ldots u_m$.

Such that
$$f(v_i) = \begin{cases} -1\,, & if\ i\ is\ odd, \\ n+1\,, & if\ i\ is\ even. \end{cases} \quad \text{for } 1 \le i \le t,$$

$$f(u_i) = \begin{cases} -1, & if\ \ f(v_i) = n+1 \\ n+1, & if\ \ f(v_i) = -1 \end{cases} \quad \text{for } 1 \le i \le m,$$

$$f(w_i) = \begin{cases} -1\,, & if\ \ f(u_i) = n+1 \\ n+1, & if\ \ f(u_i) = -1 \end{cases} \quad \text{for } 1 \le i \le m,$$

we have

$$f^*(v_i, v_{i+1}) = \begin{cases} -1 + (n+1) = n & if\ i\ is\ odd, \\ (n+1) + (-1) = n & if\ i\ is\ even. \end{cases}$$

$$f^*(v_i, u_i) = \begin{cases} -1 + (n+1) = n & if\ i\ is\ odd, \\ (n+1) + (-1) = n & if\ i\ is\ even. \end{cases}$$

$$f^*(u_i, w_i) = \begin{cases} -1 + (n+1) = n & if\ i\ is\ even, \\ (n+1) + (-1) = n & if\ i\ is\ odd. \end{cases}$$

Therefore Armed Crown graph $C_t \odot P_m$ admits n-Edge Magic Labeling when t is even.

Next, generalize the order and size of $C_t \odot P_m$ n-edge magic labeling.

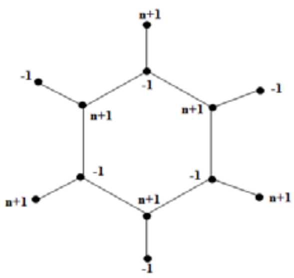
n-Edge Magic Labeling for Armed Crown Graph exists only when t ≥ 3, when t is even.

Path graph exists only when the vertices in the graph is m ≥ 2.

Let fix t = 6

Case 1:

If t = 6 and m = 2



Armed Crown Graph $C_6 \odot P_2$ exists 12 vertices and 12 edges.

Therefore $C_t \odot P_m$ exists n-Edge Magic Labeling (since t = 6 and m = 2).

Case 2:

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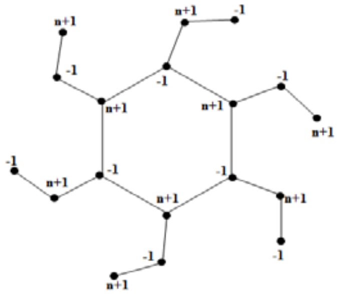
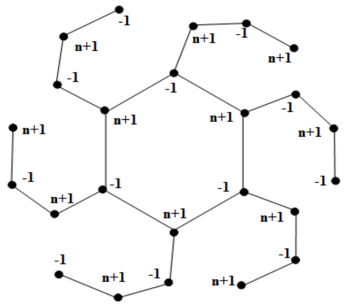
If t = 6 and m = 3



Armed Crown Graph $C_6 \odot P_3$ exists 18 vertices and 18 edges.
Therefore $C_t \odot P_m$ exists n-Edge Magic Labeling (since t = 6 and m = 3).

Case 3:

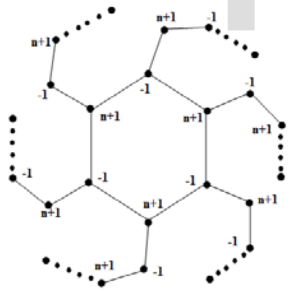
If t = 6 and m = 4



Armed Crown Graph $C_6 \odot P_4$ exists 24 vertices and 24 edges.
Therefore $C_t \odot P_m$ exists n-Edge Magic Labeling (since t = 6 and m = 4).
Continuing this process, we get

Case n:

If t = 6 and m = n



Therefore Armed Crown Graph $C_t \odot P_m$ is in the form of t = 6n (where n = 1) and m = n (for all n = 1,2,3,…..n).

Theorem 7 Let G = $P_{t+1} \odot K_{1,t}$ be a graph then G admits n-Edge Magic Labeling then graph is in the form of **2t+1 vertices** with **2t edges** (for all t = 1,2,3,…n).

Proof

Let G = (V, E) be $P_{t+1} \odot K_{1,t}$ graph. Let u_i's are vertices and v_i's are n pendent vertices to u_1. Let f: V → { -1, n + 1 }

Such that
$$f(u_i) = \begin{cases} -1\,, & if\ i\ is\ odd, \\ n+1\,, & if\ i\ is\ even. \end{cases}$$
$$f(v_i) = n+1 \quad \text{for} \quad 1 \leq i \leq t,$$

we have
$$f^*(u_i u_{i+1}) = \begin{cases} -1 + (n+1) = n & if\ i\ is\ odd, \\ (n+1) + (-1) = n & if\ i\ is\ even. \end{cases}$$
$$f^*(u_1.v_i) = -1 + (n+1) = n \quad \text{for} \quad 1 \leq i \leq t.$$

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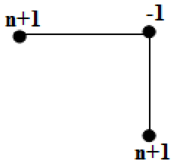
Hence $P_{t+1} \odot K_{1,t}$ be a graph then G admits n-Edge Magic Labeling.
Next, define order and size of $P_{t+1} \odot K_{1,t}$ n-edge magic labeling by method of induction.

Generalised form of $P_{t+1} \odot K_{1,t}$ graph.

Path graph only when the vertices in the graph $t \geq 2$.

Case 1:

If t = 1



$P_2 \odot K_{1,1}$ graph exists 3 vertices and 2 edges.

Let $u_1 = -1$, $u_2 = n + 1$, $v_1 = n + 1$

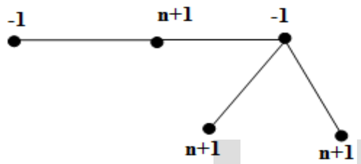
Where u_1 and v_1, u_1 and u_2 are adjacent to each other.

$$f^*(u_1 v_1) = -1 + (n + 1) = n,$$
$$f^*(u_1 u_2) = -1 + (n + 1) = n.$$

Therefore n-Edge Magic Labeling for $P_{t+1} \odot K_{1,t}$ exists 3 vertices and 2 edges Since (t = 1).

Case 2:

If t = 2

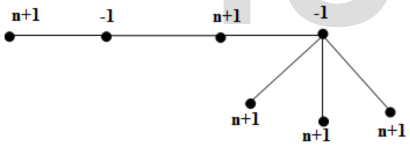


$P_3 \odot K_{1,2}$ graph exists 5 vertices and 4 edges.

Therefore n-Edge Magic Labeling for $P_{t+1} \odot K_{1,t}$ exists 5 vertices and 4 edges Since (t = 2).

Case 3:

If t = 3



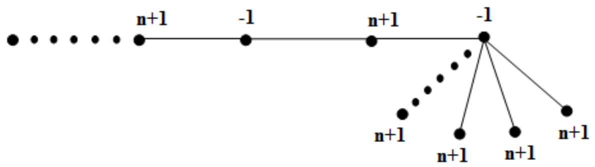
$P_4 \odot K_{1,3}$ graph exists 7 vertices and 6 edges.

Therefore n-Edge Magic Labeling for $P_{t+1} \odot K_{1,t}$ exists 7 vertices and 6 edges Since (t = 3).

Continuing this process, we get

Case n:

If t = n



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